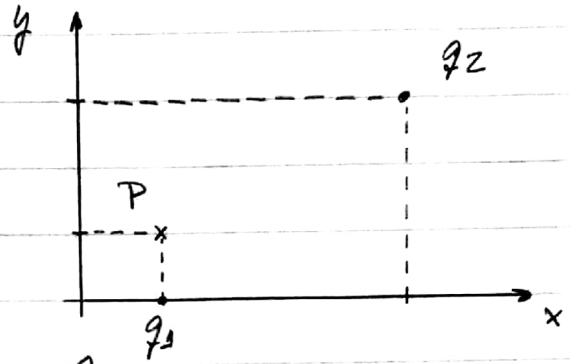
Aulão Física 3 - TVC I

1. $q_1 = 1{,}0 \cdot 10^{-12}\ C$ em $x_1 = 1{,}0\ m$, $y_1 = z_1 = 0$
$q_2 = 25{,}0\ 10^{-12}\ C$ em $x_2 = 4{,}0\ m$, $y_2 = 5{,}0\ m$ e $z_2 = 0$

$$\vec{E} = K \cdot \frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3} \cdot \vec{r}$$



$$\vec{E_1} = K \cdot \frac{q_1}{|\vec{r_p} - \vec{r_1}|} \cdot (\vec{r_p} - \vec{r_1})$$

$$\vec{r_p} - \vec{r_1} = (0, 1, 0) \Rightarrow |\vec{r_p} - \vec{r_1}| = 1\ cm$$

$$\vec{E_1} = 9 \cdot 10^9 \ \frac{10^{-12}}{(10^{-2})^3} \cdot 10^{-2}\,\hat{y} \Rightarrow \vec{E_1} = 90\ \frac{N}{C}\,\hat{y}$$

$$\vec{r_p} - \vec{r_2} = (-3, -4, 0) \Rightarrow |\vec{r_p} - \vec{r_2}| = \sqrt{3^2 + 4^2} = 5\ cm$$

$$\vec{E_2} = 9 \cdot 10^9 \cdot \frac{25 \cdot 10^{-12}}{(5 \cdot 10^{-2})^3}\ (-3, -4, 0) \cdot 10^{-2}$$

$$\vec{E_2} = 90\left(-\frac{3}{5}\hat{x} - \frac{4}{5}\hat{y}\right)\frac{N}{C}$$

Assim,

$$\vec{E_p} = \vec{E_1} + \vec{E_2}$$

$$\boxed{\vec{E_p} = (-54\hat{x} + 18\hat{y})\ \frac{N}{C}}$$

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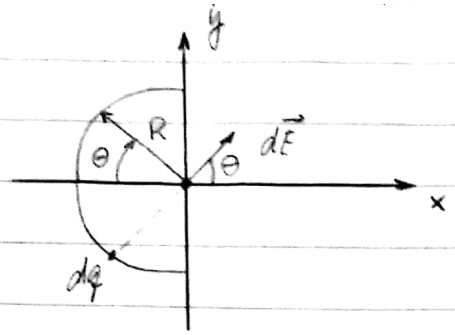
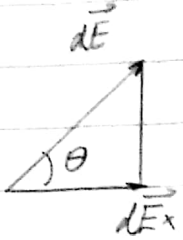
2.





$\lambda(\theta) = \lambda_0 \cdot \cos\theta$, $\lambda > 0$ $\qquad \vec{E} = K \cdot \frac{q}{r^2}\,\hat{r}$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y$$

$$\vec{E} = \int d\vec{E}_x + \int d\vec{E}_y$$

$\vec{E} = \int d\vec{E}_x$, pois $\int d\vec{E}_y = 0$ por simetria.

$$\cos\theta = \frac{d\vec{E}_x}{d\vec{E}} \Rightarrow d\vec{E}_x = d\vec{E} \cdot \cos\theta$$

$$d\vec{E} = K \cdot \frac{dq}{|r - r'|^3} \cdot (r - r')$$

$$\lambda = \frac{dq}{dl} \Rightarrow dq = \lambda \cdot dl \Rightarrow dq = \lambda \cdot r \cdot d\theta$$

$$dq = \lambda_0 \cdot r \cdot \cos\theta \cdot d\theta$$

$$d\vec{E}_x = K \cdot \frac{\lambda_0 \cdot R \cdot \cos\theta \cdot d\theta}{R^2} \cdot \cos\theta$$

$$\vec{E} = \frac{K \cdot \lambda_0}{R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$\boxed{\vec{E} = \frac{K \cdot \lambda_0 \cdot \pi}{2 \cdot R}\,\hat{x}}$$

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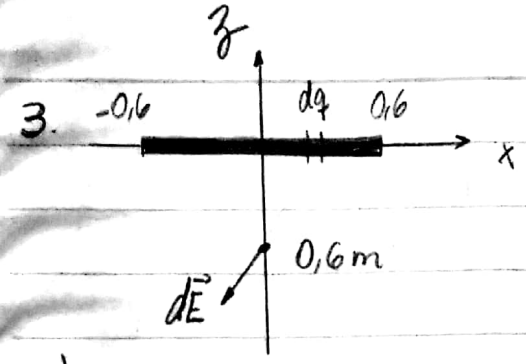
3.



a)

$Q_T = 2{,}5\,\mu C \Rightarrow \lambda = \frac{Q_T}{d} = 2{,}083\,\mu C/m \qquad dq = \lambda \cdot dx$

$$d\vec{E} = \frac{K \cdot dq}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}') \qquad \vec{r} = z\hat{z} \quad \vec{r}' = x\hat{x} \quad \vec{r}-\vec{r}' = z\hat{z} - x\hat{x}$$

$$|\vec{r}-\vec{r}'| = \sqrt{z^2+x^2}$$

$$d\vec{E} = \frac{K\lambda \cdot dx}{(z^2+x^2)^{3/2}}(z\hat{z} - x\hat{x})$$

Por simetria, o campo elétrico tem apenas componente na direção $\hat{z}$:

$$\vec{E} = \int_{-a}^{a} \frac{k \cdot \lambda \cdot dx}{(z^2+x^2)^{3/2}} z\hat{z} = K \cdot \lambda \cdot z \cdot \hat{z} \int_{-a}^{a} \frac{1}{(z^2+x^2)^{3/2}} dx$$

$$\vec{E} = K \cdot \lambda z \hat{z} \frac{2 \cdot a}{z^2\sqrt{a^2+z^2}}$$

$$\vec{E} = \frac{2K\lambda a}{z\sqrt{a^2+z^2}} \cdot \hat{z}$$

Substituindo: $\vec{E}_+ = -44187{,}1\ N/C\ \hat{z}$

A outra barra produz $\vec{E}_- = -44187{,}1\ N/C\ \hat{x}$

$$\vec{E}_p = \vec{E}_+ + \vec{E}_- = 44187{,}1\,(-\hat{x} - \hat{z})\ N/C$$

$$|\vec{E}_p| = 6{,}25\,10^4\ N/C$$

b) $\vec{F} = 1{,}6 \cdot 10^{-19}\ 44187{,}1\,(\hat{x}+\hat{z})\ N$

$\vec{F} = 7{,}07\,(\hat{x}+\hat{z})\ N$

$|\vec{F}| = 9{,}9984 \cdot 10^{-15}\ N$

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4. $\rho(r) = \rho_0 \cdot \frac{a}{r} e^{-\frac{r^2}{2a^2}}$

a) $Q_{TOTAL} = \iiint_V \rho_0 \cdot \frac{a}{r} \cdot e^{-\frac{r^2}{2a^2}} \, dV$

$$Q_T = \int_0^{2\pi} \int_0^{\pi} \int_0^{+\infty} \rho_0 \, a \cdot e^{-\frac{r^2}{2a^2}} \cdot r \operatorname{sen}\varphi \, dr \, d\varphi \, d\theta$$

$$Q_T = 2\pi \cdot \rho_0 \cdot a \cdot \int_0^{\pi} \int_0^{+\infty} \operatorname{sen}\varphi \cdot e^{-\frac{r^2}{2a^2}} \cdot r \, dr \, d\varphi$$

$$Q_T = 2\pi \rho_0 \, a \cdot \left(+a^2 \cdot e^{-\frac{r^2}{2a^2}}\right)\Big|_{+\infty}^{0} \cdot \cos\varphi \Big|_{\pi}^{0}$$

$$Q_T = 2\pi \rho_0 \cdot a \cdot a^2 \cdot 2$$

$$\boxed{Q_T = 4\pi \rho_0 a^3}$$

b) $\oiint \vec{E} \cdot d\vec{A} = \frac{q_{int}}{\varepsilon_0}$

$$\oiint_S E \, \hat{r} \, dA \cdot \hat{r} = \frac{1}{\varepsilon_0} \cdot \int_0^{2\pi} \int_0^{\pi} \int_0^{r} \rho(r) \, dV$$

$$E(r) \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \cdot 2\pi \rho_0 \cdot a \cdot 2 \cdot a^2 \cdot e^{-\frac{r^2}{2a^2}}$$

$$E(r) = \frac{4\pi \rho_0 \cdot a^3 \cdot e^{-\frac{r^2}{2a^2}}}{\varepsilon_0 \cdot 4\pi r^2}$$

$$\boxed{\vec{E}(r) = \frac{\rho_0 \cdot a^3 \cdot e^{-\frac{r^2}{2a^2}}}{\varepsilon_0 \cdot r^2} \hat{r}}$$

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5a) $\sigma \cdot dA = dq$

$\sigma \cdot r \cdot dr \cdot d\theta = dq$

$$dV = K \cdot \frac{\sigma \cdot r \cdot dr d\theta}{\sqrt{r^2 + z^2}}$$

$$V = \iint_A dV = \int_0^{2\pi}\int_0^{R} K \cdot \frac{\sigma \cdot r}{\sqrt{r^2+z^2}}\, dr d\theta$$

$$V = K \cdot \sigma \int_0^{2\pi}\int_0^{R} \frac{r}{\sqrt{r^2+z^2}}\, dr d\theta$$

$$V = K \cdot \sigma \cdot 2\pi \cdot \sqrt{a^2+z^2}\Big|_0^R$$

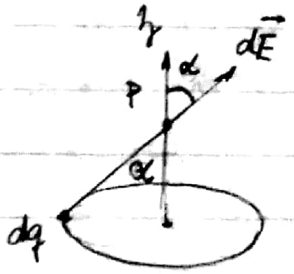
$$V = K \cdot \sigma \cdot 2\pi \sqrt{R^2+z^2}$$

b) Como $V(\infty) = 0$

$$U = V \cdot q$$

$$U = q_0 \cdot K \sigma \cdot 2\pi \sqrt{R^2+z^2}$$

6a)



$$\vec{E} = \frac{\lambda \cdot r \cdot z \cdot \hat{K}}{2\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$\lambda = \frac{dq}{dl} = \frac{dq}{r\,d\theta} \Rightarrow dq = \lambda \cdot r \cdot d\theta$$

Por simetria, pode-se ver que há campo elétrico apenas na direção $\hat{K}$, assim:

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$$d\vec{E}_z = dE \cos\alpha \, \hat{k}$$

$$d\vec{E}_z = K \cdot \frac{dq}{(\sqrt{r^2+z^2})^2} \cos\alpha \cdot \hat{k}$$

$$d\vec{E}_z = K \cdot \frac{\lambda \cdot r \cdot d\theta}{(\sqrt{r^2+z^2})^2} \cdot \frac{z}{\sqrt{r^2+z^2}} \cdot \hat{k}$$

$$\vec{E} = \int_0^{2\pi} d\vec{E}_z = \frac{K \cdot \lambda \cdot r \cdot 2\pi \cdot z}{(r^2+z^2)^{3/2}} \cdot \hat{k}$$

Como $K = \frac{1}{4\pi\varepsilon_0}$

$$\boxed{\vec{E} = \frac{\lambda \cdot r \cdot z \cdot \hat{k}}{2\varepsilon_0 (r^2+z^2)^{3/2}}}$$

b) $$\lim_{r\to 0} \vec{E}(r) = \frac{\lambda \cdot z \cdot \hat{k}}{2 \cdot \varepsilon_0} \lim_{r\to 0} \frac{r}{(r^2+z^2)^{3/2}} = 0$$

$$\lim_{r\to +\infty} \vec{E}(r) = \frac{\lambda \cdot z \cdot \hat{k}}{2\varepsilon_0} \lim_{r\to \infty} \frac{r}{(r^2+z^2)^{3/2}} = 0$$

Para $r \to +\infty$, todas as cargas estão longe o suficiente a ponto de o campo elétrico produzido por elas não influenciarem

Para $r \to 0$, por simetria, pode-se ver que o campo elétrico de uma carga é cancelada pela sua oposta

c) $$|\vec{E}| = \frac{\lambda z}{2 \cdot \varepsilon_0} \cdot \frac{r}{(r^2+z^2)^{3/2}}$$

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$$\frac{d|\vec{E}|}{dr} = \frac{\lambda z}{2\varepsilon_0} \cdot \frac{(r^2+z^2)^{3/2} - r \cdot \frac{3}{2}(r^2+z^2)^{1/2} \cdot 2r}{(r^2+z^2)^3} = 0$$

$$(r^2+z^2)^{3/2} - 3r^2(r^2+z^2)^{1/2} = 0$$

$$3r^2 = (r^2+z^2)$$

$$2r^2 = z^2$$

$$\text{para } r > 0 \Rightarrow r = \frac{z}{\sqrt{2}}$$

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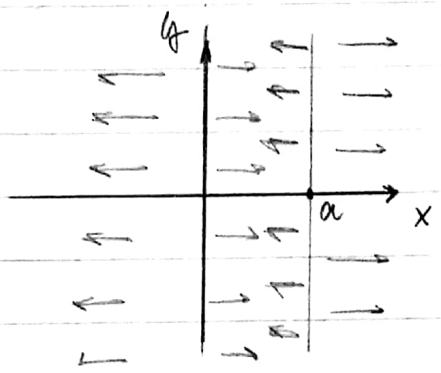

Densidade $+\sigma$ para ambos planos.

$$\oiint \vec{E} \cdot d\vec{S} = \frac{q_{int}}{\varepsilon_0}$$

$$2 \cdot E \cdot \pi \cdot r^2 = \frac{\sigma \cdot r^2 \pi}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

O campo elétrico nas regiões será dado por:

$$E(x) = \begin{cases} \frac{\sigma}{\varepsilon_0}(-\hat{x}), & x < 0 \\ 0, & 0 < x < a \\ \frac{\sigma}{\varepsilon_0}(\hat{x}), & x > a \end{cases}$$

Como o campo elétrico produzido pelas placas é constante, a potencial elétrico irá depender da posição x.

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Assim, fazendo $V(0) = 0$,

$$V(x) - V(0) = \int_0^x \vec{E} \cdot d\vec{\ell}$$

Para $x < 0$:

$$V(x) = \int_0^x \frac{\sigma}{\varepsilon_0}(-\hat{x})\, dx \cdot (+\hat{x})$$

$$V(x) = -\frac{\sigma}{\varepsilon_0}\int_0^x d\ell$$

$$V(x) = -\frac{\sigma \cdot x}{\varepsilon_0}$$

Para $0 < x < a$:

Como $\vec{E} = \vec{0}$, $V(x) = 0$.

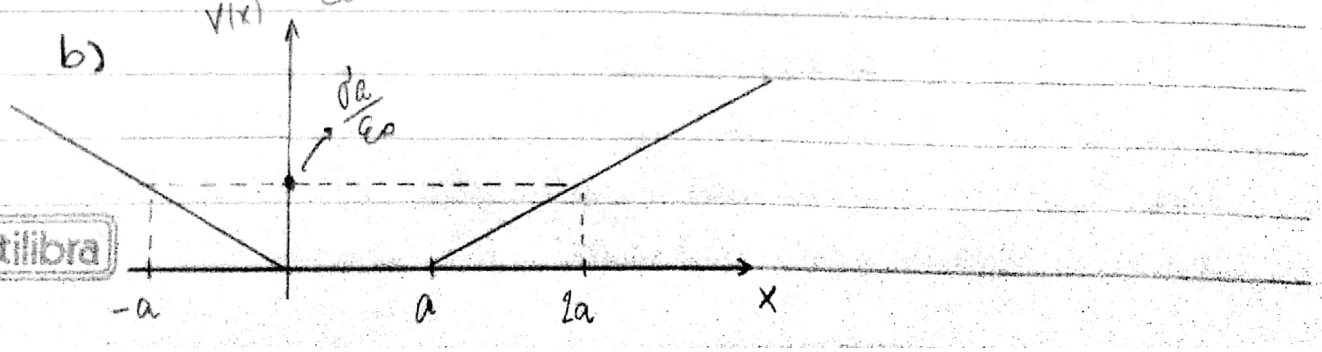
Para $x > a$:

$$V(x) = \int_0^x \vec{E} \cdot d\vec{\ell} = \int_0^a \vec{E} \cdot d\vec{\ell} + \int_a^x \frac{\sigma}{\varepsilon_0} \hat{x}\, dx\, \hat{x}$$

$$V(x) = \frac{\sigma}{\varepsilon_0}(x - a)$$

$$V(x) \begin{cases} -\frac{\sigma x}{\varepsilon_0}, & x < 0 \\ 0, & 0 < x < a \\ \frac{\sigma}{\varepsilon_0}(x-a), & x > a \end{cases}$$

b)



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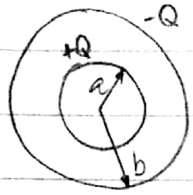
c) $V(x) - V(\infty) = \int_{+\infty}^{x} \frac{\sigma'}{\varepsilon_0} \hat{x} \cdot dx \cdot \hat{x} = \int_{+\infty}^{x} \frac{\sigma}{\varepsilon_0} dx = -\infty$

Para esse referencial, $V(x)$ não existe.

8.



a) Utilizando a lei de Gauss:

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{int}}{\varepsilon_0}$$

(i) para $r < a$:

$$\oiint_S E \cdot \hat{r} \cdot dA \cdot \hat{r} = \frac{q_{int}(r)}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q \cdot r^3}{\varepsilon_0 \cdot a^3}$$

$$\vec{E}(r) = \frac{Q \cdot r}{4\pi\varepsilon_0 \cdot a^3} \hat{r}$$

(ii) para $a \leq r < b$:

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{r}$$

(iii) para $r > b$:

$$\vec{E}(r) = \vec{0}$$

$$\gamma = \frac{dq}{dV} = \frac{dq}{r^2 \operatorname{sen}\varphi \, dr\, d\varphi\, d\theta}$$

$$dq = \gamma \cdot r^2 \operatorname{sen}\varphi \, dr\, d\varphi\, d\theta$$

$$q_{int} = \iiint_V \gamma r^2 \operatorname{sen}\varphi \, dr\, d\varphi\, d\theta$$

$$q_{int}(r) = \int_0^{2\pi} \int_0^{\pi} \int_0^{r} \gamma r'^2 \operatorname{sen}\varphi \, dr'\, d\varphi\, d\theta$$

$$q_{int}(r) = 2\pi \cdot \gamma \frac{r^3}{3} \cdot 2$$

$$q_{int}(r) = \frac{4}{3}\pi r^3 \cdot \frac{Q}{\frac{4}{3}\pi a^3}$$

$$q_{int}(r) = \frac{r^3}{a^3} \cdot Q$$

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b) $Q = C \cdot V$

$$V_{ab} = \int_a^b \frac{Q}{4\pi\varepsilon_0 \cdot r^2}\, dr$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2}\, dr = \frac{Q}{4\pi\varepsilon_0} \left.\frac{1}{r}\right|_b^a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$Q = C \cdot \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

c)

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{6371000}}$$

$$C_T = 7{,}085 \cdot 10^{-4}\ F$$

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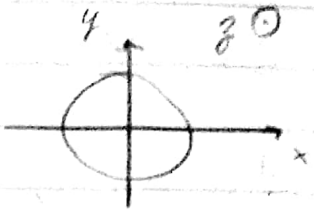
Aulão Física 3 - TVC II

1. $\vec{j}(r) = Ar^2\hat{k}$, para $0 < r < R$



a) Encontrando a corrente total

$$I = \iint_S \vec{j} \cdot d\vec{A} = \iint_S Ar^2\hat{k} \cdot dA \cdot \hat{k}$$

$$I = \int_0^{2\pi}\int_0^{R} Ar^2 \cdot r \, dr\, d\theta$$

$$I = 2\pi \cdot A \left.\frac{r^4}{4}\right|_0^R$$

$$I = \frac{R^4 \cdot \pi \cdot A}{2}$$

Como a corrente tem sentido para fora da página, pela regra da mão direita, o campo magnético tem sentido anti-horário.

b) Para $0 < r < R$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{int}$$

$$\int_0^{2\pi} B \cdot \hat{\theta} \cdot r \, d\theta \, \hat{\theta} = \mu_0 \cdot \int_0^{2\pi}\int_0^{r} Ar'^2\hat{k} \cdot r' \, dr' d\theta \, \hat{k}$$

$$B \cdot 2\pi r = \mu_0 \cdot A \, 2\pi \frac{r^4}{4}$$

$$\boxed{\vec{B} = \frac{A \cdot \mu_0 \cdot r^3}{4}\hat{\theta}}$$

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Para $r > R$:

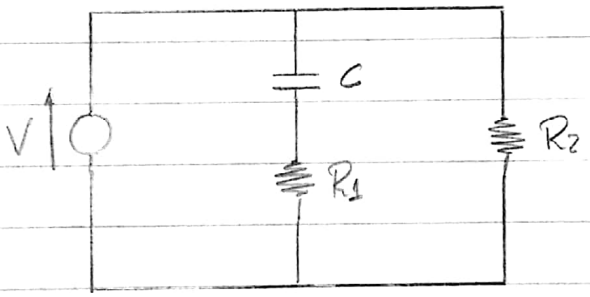
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \, I_{int} = \mu_0 \iint \vec{j}(r) \, d\vec{A}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_0^{2\pi} \int_0^R A r^2 \cdot r \, dr \, d\theta + \mu_0 \int_0^{2\pi} \int_R^r j(r) \, r \, dr \, d\theta$$

$$B \cdot 2\pi r = \mu_0 \cdot \frac{R^4 \cdot \pi \cdot A}{2}$$

$$\boxed{\vec{B} = \frac{\mu_0 R^4 \cdot A}{4r} \hat{\theta}}$$

2.



Em um circuito RC, a tensão no capacitor é dada por:

$$V_c(t) = V(1 - e^{-t/RC})$$

$V_c(0+) = 0 \Rightarrow$ curto circuito

$V_c(+\infty) = V \Rightarrow$ circuito aberto

a) Redesenhando o circuito para o tempo 0+:

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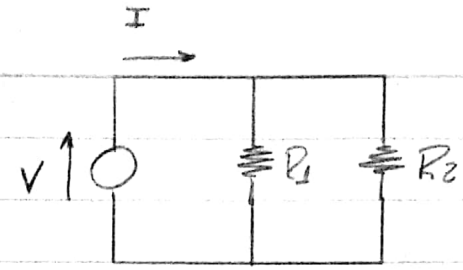
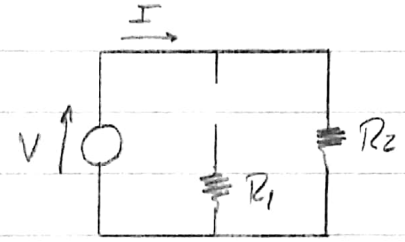


$$I = \frac{V}{Req}$$

$$Req = \frac{R_1 R_2}{R_1 + R_2}$$

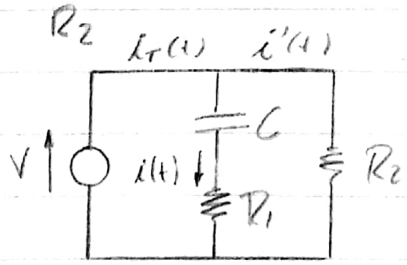
$$I = \frac{V(R_1 + R_2)}{R_1 \cdot R_2}$$

b) Redesenhando o circuito



$$I = \frac{V}{R_2}$$

c)



$R \equiv R_1$

$$V = R_1 \cdot i + \frac{1}{C}\int i\,dt \Rightarrow V = R\frac{dq}{dt} + \frac{q}{C}$$

$$q' + \frac{1}{RC}q = \frac{V}{R}$$

$$q' \cdot e^{\frac{t}{RC}} + \frac{1}{RC} q \cdot e^{\frac{t}{RC}} = \frac{V}{R} \cdot e^{\frac{t}{RC}}$$

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$$\frac{d}{dt}\left(e^{\frac{t}{RC}} \cdot q\right) = \frac{V}{R} \cdot e^{\frac{t}{RC}}$$

$$e^{t/RC}\, q = \frac{V}{R} \cdot RC \cdot e^{t/RC} + K$$

$$q(t) = VC + K \cdot e^{-t/RC}$$

Capacitor inicialmente descarregado: $q(0+) = 0$

$$q(0+) = 0 = VC + K$$

$$K = -VC$$

$$q(t) = CV\left(1 - e^{-t/R_1C}\right)$$

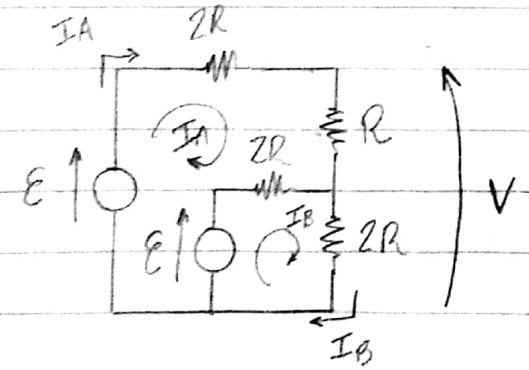
a) $i_T(t) = i(t) + i'(t)$

$$i_T(t) = \frac{dq}{dt} + \frac{V}{R_2}$$

$$\frac{dq}{dt} = CV \cdot \frac{1}{R_1C} \cdot e^{-t/R_1C} = \frac{V}{R_1} e^{-t/R_1C}$$

$$i_T(t) = \frac{V}{R_1} e^{-t/R_1C} + \frac{V}{R_2}$$

3.



a) Pela lei das malhas:

$$\mathcal{E} - \mathcal{E} = 2RI_A + RI_A + 2R(I_A - I_B)$$

$$\mathcal{E} = 2R(I_B - I_A) + 2RI_B$$

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$$\begin{cases} 5R I_A - 2R I_B = 0 \\ -2R I_A + 4R I_B = \mathcal{E} \end{cases}$$

$$I_A = \mathcal{E}/8R$$

$$I_B = 5\mathcal{E}/16R$$

b) $V = R \cdot I_A + 2R \cdot I_B$

$V = R \cdot \mathcal{E}/8R + 2R \cdot 5\mathcal{E}/16R$

$V = \mathcal{E}/8 + 10\mathcal{E}/16$

$V = 20/8 + 10 \cdot 20/16$

$\boxed{V = 15\,V}$

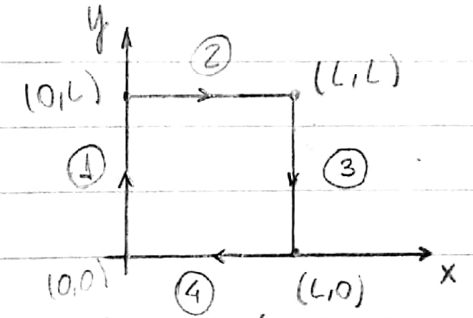
0 1 0

1 0 0

(0 0 1)

4. $d\vec{F} = i\, d\vec{\ell} \times \vec{B}$

a)



① $\vec{F} = \int d\vec{F} = \int i\, d\vec{\ell} \times \vec{B}$

$\vec{F} = \int_L^L i\, dy \cdot \hat{y} \times \left(\frac{B_0 y}{L} \hat{x} + \frac{B_0 x}{L} \hat{y} \right)$

$\vec{F} = i \cdot \frac{B_0}{L} \int_0^L (\hat{y} \times (y\hat{x} + x\hat{y})\, dy$

$\vec{F} = i \frac{B_0}{L} \int_0^L -\hat{z}\, y\, dy$

$\vec{F}_1 = -\frac{B_0 i}{L} \hat{z} \cdot \frac{y^2}{2} \Big|_0^L$

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$$\vec{F}_1 = \frac{B_0 \cdot i \cdot L}{2}(-\hat{z})$$

③ Por simetria $\vec{F}_3 = \frac{B_0 \cdot i \cdot L}{2}\hat{z}$

② $$\vec{F}_2 = \int_0^L i\,dx\,\hat{x} \times \left(\frac{B_0 y}{L}\hat{x} + \frac{B_0 x}{L}\hat{y}\right)$$

$$\vec{F}_2 = i\,\frac{B_0}{L}\int_0^L (\hat{x} \times (y\hat{x} + x\hat{y})\,dx$$

$$\vec{F}_2 = \frac{B_0 i}{L}\int_0^L x\,\hat{z}\,dx$$

$$\vec{F}_2 = \frac{B_0 i}{L}\cdot\hat{z}\,\frac{x^2}{2}\Big|_0^L = \frac{B_0 i}{L}\,\hat{z}\,\frac{L^2}{2}$$

$$\vec{F}_2 = \frac{1}{2}\cdot B_0 \cdot i \cdot L \cdot \hat{z}$$

④ Por simetria $\vec{F}_4 = \frac{1}{2} B_0 \cdot i \cdot L(-\hat{z})$

b) $\vec{F}_T = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$

c) $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = (L\hat{y}) \times \left(\frac{1}{2} B_0 \cdot i \cdot L\,\hat{z}\right)$$

$$\vec{\tau} = \frac{1}{2} B_0 \cdot i \cdot L^2\,\hat{x}$$

d) Não, pois o campo magnético não é constante ao longo de toda a espira.

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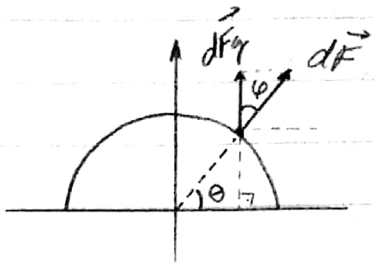
5. $\vec{B} = B\hat{k}$



$$\vec{dl} = dl\,\hat{\theta} = r\,d\theta\,\hat{\theta}$$

$$\vec{F} = \int_C i\,(r\,d\theta\,\hat{\theta}) \times (B\hat{k})$$

$$\vec{F} = i \cdot r \cdot B \cdot \int_0^{2\pi} (\hat{\theta} \times \hat{k})\,d\theta$$

$$\vec{F} = i \cdot r \cdot 2\pi \cdot B\,\hat{r} \Rightarrow \text{Errado}$$

$$d\vec{F} = i\,r\,d\theta\,\hat{\theta} \times B\,\hat{k}$$

$$d\vec{F} = i\,r\,B\,\hat{r}\,d\theta \Rightarrow dF = iBr\,d\theta$$

Por simetria, o campo magnético tem apenas componente $\hat{y}$.

$$\cos\varphi = \frac{d\vec{F}_y}{d\vec{F}}$$

$$d\vec{F}_y = dF \cdot \cos\varphi\,\hat{y} = dF \cdot \cos\left(\frac{\pi}{2} - \theta\right)\hat{y}$$

$$\vec{F}_T = \int_C d\vec{F}_y = \int_0^{\pi} i \cdot r \cdot B\,d\theta \cdot \cos\left(\frac{\pi}{2} - \theta\right)\hat{y}$$

$$\vec{F}_T = i \cdot r \cdot B \cdot \hat{y} \int_0^{\pi} \cos\left(\frac{\pi}{2} - \theta\right) d\theta$$

$$\vec{F}_T = 2\,Bri\,\hat{y}$$

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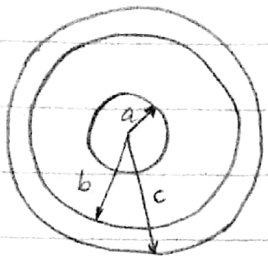
6.



a)

Lei de Ampère: $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{env}$

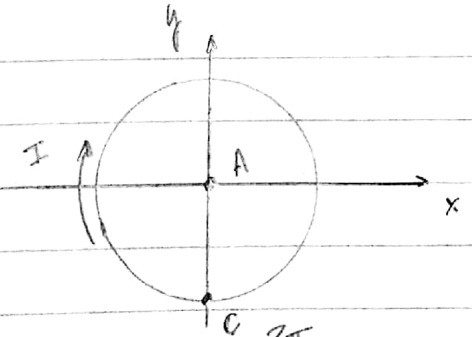
$B \cdot 2\pi r = \mu_0 \cdot I$

$B = \frac{\mu_0 I}{2\pi r}$

b) Como $I = 0$, $\vec{B} = \vec{0}$.

7a) Lei de Biot-Savart:

$$\vec{B} = \frac{\mu_0}{4\pi} \int_C \frac{I d\vec{l} \times \hat{r}}{r^2}$$



$d\vec{l} = r\,d\theta\,(-\hat{\theta})$

$\hat{r}' = -\hat{r}$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{r\,d\theta\,(-\hat{\theta}) \times (-\hat{r})}{r^2}$$

Pela regra da mão direita em um elemento diferencial de corrente, a direção do campo magnético é $-\hat{z} = \hat{\theta} \times \hat{r}$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{(-\hat{z})}{r} \cdot \int_0^{2\pi} d\theta$$

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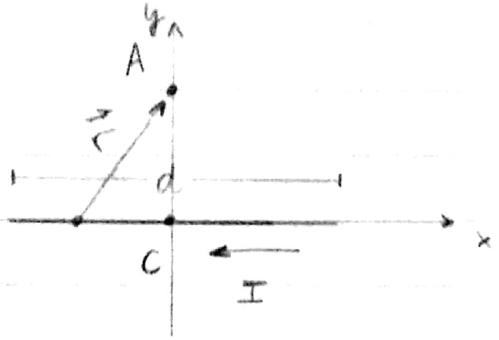
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\pi}{r} \cdot (-\hat{z}) \cdot I = \mu_0 I / D$$

b)



$$R = \frac{D}{2}$$

$$\alpha = 2\pi R \qquad d\vec{\ell} = dx(-\hat{x})$$

$$\alpha = D\pi \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{(0,R)-(x,0)}{|(0,R)-(x,0)|} = \frac{-x\hat{x} + R\hat{y}}{\sqrt{x^2+R^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_C I \frac{dx(-\hat{x}) \times \frac{-x\hat{x}+R\hat{y}}{\sqrt{x^2+R^2}}}{(\sqrt{x^2+R^2})^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_{-\frac{D\pi}{2}}^{\frac{D\pi}{2}} \frac{R\,dx}{(x^2+R^2)^{3/2}} (-\hat{z})$$

$$\vec{B} = \frac{\mu_0 \, I \cdot R}{4\pi} (-\hat{z}) \cdot \int_{-\frac{D\pi}{2}}^{\frac{D\pi}{2}} \frac{dx}{(x^2+R^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 \, I \, R (-\hat{z})}{4\pi} \left. \frac{x}{R^2\sqrt{R^2+x^2}} \right|_{-\frac{D\pi}{2}}^{\frac{D\pi}{2}}$$

$$\vec{B} = \frac{\mu_0 \, I}{D\sqrt{1+\pi}} (-\hat{z})$$

c) O campo produzido pela barra reta é menor

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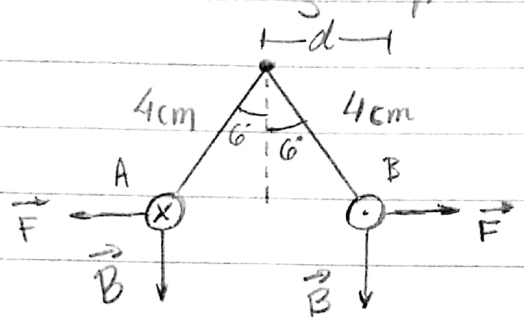
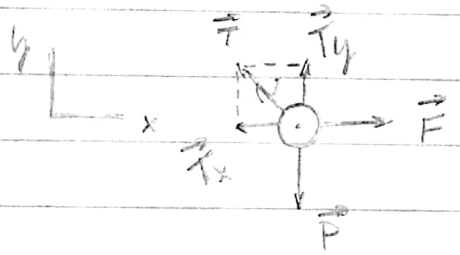
8. Pela lei de ampère, o campo produzido por uma barra infinita é:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I$$

$$B = \frac{\mu_0 \cdot I}{2\pi r}$$

Olhando a situação por outro ângulo:





Considerando 1m de barra:

$$\vec{P} = 0{,}175\,(-\hat{y})\ N$$

$$\vec{F} = i\,\vec{E} \times \vec{l}$$

$$\vec{F} = i \cdot \frac{\mu_0 \cdot i}{2\pi r} \cdot 1m\ (\hat{x})$$

$$\sin 6° = \frac{d}{0{,}04} \Rightarrow r = 2d = 0{,}08 \cdot \sin 6°$$

$$\vec{F} = \frac{\mu_0 \cdot I^2}{2\pi \cdot 0{,}08 \cdot \sin 6°}\ (\hat{x})$$

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$$T_y = P = 0{,}125\ N$$

$$\operatorname{tg} 6° = \frac{T_x}{T_y}$$

$$T_x = \operatorname{tg} 6° \cdot 0{,}125$$

$$\operatorname{tg} 6° \cdot 0{,}125 = \frac{\mu_0 \cdot I^2}{2\pi \cdot 0{,}08 \cdot \operatorname{sen} 6°}$$

$$I = 23{,}44\ A$$

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